Finite element modeling and experimental study on bending and vibration of laminated stiffened glass fiber/polyester composite plates

Tran Ich Thinh *, Tran Huu Quoc
School of Mechanical Engineering, Hanoi University of Science and Technology, Viet Nam

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A B S T R A C T

In this paper, free vibration and bending failure of laminated stiffened glass fiber/polyester composite plates with laminated open section (rectangular or T-shaped) and closed section (hat shaped) of stiffeners have been studied by finite element method and experiment. A 9-noded isoparametric element with 9 degrees of freedom per node is developed for the plates. The stiffener element is a 3-noded isoparametric beam element with 5 degrees of freedom per node and the stiffeners can be positioned anywhere within the plate element. The natural frequencies of the laminated stiffened plates are determined experimentally by Dewebook device and DasyLab 7.0 software. The results calculated by computational model for above plates under different boundary conditions are in good agreement with experiments. The failure problems of these stiffened glass fiber/polyester composite plates are also investigated.

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1. Introduction

Laminated stiffened composite plates are extensively used in aircraft structures, ship and boat hulls, bridge decks and other industrial applications due to their high stiffness–weight ratio and high strength weight ratio. Stiffeners can achieve greater strength with relatively less material, which improves the strength/weight ratio and makes the structure cost efficient. Consequently, many sophisticated models and methods (grillage methods, orthotropic model, Rayleigh–Ritz method, finite difference method, semi-analytic finite different method and finite element method) have been developed over the years for an appropriate structural analysis for these plates. Among them, finite element method has widely used because it has been found to be reasonably accurate with less complexity to model stiffened plates.

Chattopadhyay et al. [1] have developed a finite element method based on first-order shear deformation theory to analyze free vibration behaviour of composite stiffened plates. An 8-noded isoparametric quadratic stiffened plate bending element has been used in their formulation. The same shape function is used for the shell and stiffener element. The formulation restricts itself to model stiffeners which are required to pass through two adjacent edges of the plate element.

The FE model of Kolli and Chandrashekhara [2] consists of the 9-noded rectangular plate element and 3-noded beam element. The formulation requires the nodal lines of the plate elements to pass through the stiffener. Therefore, arbitrary oriented stiffeners cannot be analyzed using this formulation. Only open section stiffeners can be modeled by this formulation.

Edward Sadeck and Samer Tawfik [3] used a stiffened plate element that is composed of a rectangular 9-noded rectangular plate element and a number of 3-noded stiffener elements placed within the plate element and parallel to the element edges. The existing finite element techniques stimulate the stiffener based on Reissner–Mindlin theory. In [6] Satish Kumar and Mukhopadhyay used a new finite element for buckling analysis of laminated stiffened plates, the model based on first-order shear deformation theory so both the plate element and the stiffener element include transverse shear deformation based on Reissner–Mindlin theory.

Koko and Olson [7] applied a numerical technique for vibration analysis of isotropic stiffened plates. They used a super element which is macro-element having analytical as well as the usual finite element shape functions. Peng et al. [8] applied the laminated stiffened plates. The formulation requires the nodal lines of the plate elements to pass through the stiffener. Therefore, arbitrary oriented stiffeners cannot be analyzed using this formulation.
element-free Galerkin (EFG) method to the static analysis of concentrically and eccentrically stiffened plates, the stiffeners need to be placed along the mesh lines. Akhras and Li [9] developed the progressive failure analysis for thick composite plates using the spline finite strip method based on Cho’s higher order zigzag laminate theory and Lee’s failure criterion. However, only infinitesimal strain was considered in the formulation of the spline finite strip equations.

Qing et al. [10] developed a novel mathematical model based on the semi-analytical solution of the state-vector equation theory for free vibration analysis of stiffened laminated plates. The method accounts for the compatibility of displacements and stresses on the interface between the plate and stiffeners, the transverse shear deformation, and naturally the rotary inertia of the plate and stiffeners. Meanwhile, there is no restriction on the thickness of plate and the height of stiffeners.

Prusty [11] performed a progressive failure analysis by finite element method based on first-order shear deformation theory for laminated unstiffened and stiffened composite panels under transverse loads and the failure loading was predicted by using the Tsai-Wu criterion and the total discount stiffness reduction model.

Zahari and El-Zafrany [12] developed a progressive damage analysis methodology for stress analysis of composite laminated shells using new derivations of finite strip methods based on Mindlin’s plate-bending theory, where the non-linear equations (finite strain) were derived using the tangential stiffness matrix approach, with all integrations over the plate thickness carried out analytically.

In the previous works, we have used finite element method for free vibration and bending analysis of laminated stiffened composite plates with laminated open section (rectangular and T) and closed section (hat or U) stiffeners [13]. In that paper, the natural frequencies calculated by our model were compared with those published by other authors but were not compared with experiment. In [14], we analyzed progressive failure of laminated stiffened composite plates by using total discount material properties of failed layer approach.

In this study, finite element model and experiment on free vibration and progressive failure of stiffened laminated composite plates reinforced by laminated open section and closed section stiffeners under various boundary conditions are investigated. Our model is based on a higher-order displacement theory which eliminates the need to use shear correction coefficients and make the model applicable for both thick and thin stiffened composite plates. The stiffeners with different types of section can be positioned anywhere within the plate element. The natural frequencies measured by our experiment are compared with the results calculated by our finite element model. The failure problems of these stiffened glass fiber/polyester composite plates are also investigated by using failure mode in action to discount the mechanical properties of failed layer.

2. Theoretical formulation

2.1. Displacement field

The geometry of the laminated stiffened composite plate is similar to that shown in Fig. 1. A stiffened plate is composed of a plate and a number of stiffeners placed inside the plate. Both plate and stiffeners are made up of laminated composites.

Consider a laminated composite plate which is parallel to the x–y plane. The upper and lower surfaces of the plate are defined by $z = h/2$ and $z = -h/2$, respectively, where $h$ is the thickness of the plate.
where:

\[
[m] = \begin{bmatrix}
0 & 0 & I_1 & 0 & I_2 & 0 & I_3 & 0 \\
0 & I_0 & 0 & I_1 & 0 & I_2 & 0 & I_3 \\
0 & 0 & I_0 & 0 & 0 & 0 & 0 & 0 \\
I_1 & 0 & 0 & I_2 & 0 & I_3 & 0 & I_4 \\
0 & I_0 & 0 & I_1 & 0 & I_2 & 0 & I_3 \\
I_2 & 0 & I_3 & 0 & I_4 & 0 & I_5 & 0 \\
I_3 & 0 & I_4 & 0 & I_5 & 0 & I_6 & 0 \\
I_4 & 0 & I_5 & 0 & I_6 & 0 & I_0 & 0
\end{bmatrix}
\]

The element stiffness matrix of the stiffener can be expressed in the following equation:

\[
K_{\text{stiffener}} = \int_{x_1}^{x_2} \begin{bmatrix} B_n^T & D_n & B_n \end{bmatrix} dx
\]

where:

\[
[B_n]_{7x7} = \begin{bmatrix}
A_1 & B_1 & D_1 & 0 & 0 & 0 \\
C_1 & E_1 & F_1 & 0 & 0 & 0 \\
B_2 & D_2 & E_2 & 0 & 0 & 0 \\
D_3 & F_3 & E_3 & 0 & 0 & 0 \\
0 & 0 & 0 & A_3 & B_3 & C_3 \\
0 & 0 & 0 & 0 & B_3 & C_3 & D_3 \\
0 & 0 & 0 & 0 & 0 & C_3 & D_3 & E_3
\end{bmatrix}
\]

The most difficulty in analysis of stiffened laminated composite plates is to establish the model which allows the stiffeners can be positioned anywhere within the plates and the stiffener laminations can be either parallel or perpendicular or make any angle to the laminations of the plates.

In this paper, we propose an approach for that problem as follows:

- Considering the \(\alpha\)-directional stiffener (Fig. 3a) is the basic stiffener.
- Other directional stiffeners (Fig. 3b) will be obtained by rotating \(\alpha\)-directional stiffener about \(z\)-axis by \(\gamma\) angles then about \(y\)-axis by \(\beta\) angles.

The nodal displacements of the stiffener element are transformed to the plate element nodes by using the transformation matrix as:

\[
\begin{bmatrix} U_p \\
V_p \\
W_p \end{bmatrix} = R_{yz} \begin{bmatrix} U_s \\
V_s \\
W_s \end{bmatrix}
\]

where \(R_{yz}\) is the rotation matrix about \(z\)-axis and \(y\)-axis:

\[
R_{yz} = \begin{bmatrix} c_1 c_2 & c_1 s_2 & -s_2 \\
-c_2 & c_1 & 0 \\
s_1 c_2 & s_1 s_2 & c_1 \end{bmatrix}
\]

so we have:

\[
\{q_v\} = \Lambda \{q\}
\]

\[
\{q_v\} = [u_{vp} \, w_{vp} \, \theta_{vp} \, \varphi_{vp} \, \theta_{vp} \, \varphi_{vp} \, \varphi_{vp}]^T \quad \text{and} \quad \{q\} = [u_{op} \, w_{op} \, \theta_{op} \, \varphi_{op} \, u_{op} \, w_{op} \, \theta_{op} \, \varphi_{op}]^T
\]

Since the nodes of the beam element are within the plate element, it is possible to interpolate the nodal displacements of the beam element from the plate element nodal displacements by

\[
q_{uv} = \sum_{j=1}^{3} \{\Lambda\}^{-1} C \{\tilde{q}\} = \sum_{j=1}^{3} [\Lambda]^{-1} C \{\tilde{q}\}
\]

where \([\Lambda]\) is the transformation matrix of the nodal displacements from the beam element into the plate element nodes defined as follows:

\[
[C] = \sum_{j=1}^{3} \{N\}_{i,j} \{\chi\}_{ij}
\]

\[
\text{Fig. 3. Inclination of stiffeners.}
\]
where \( N_i \) are the plate element shape functions and \([t_{ii}] \) is a nine by nine identity matrix.

And we obtain the transformation matrix \( V_{(15 \times 81)} = \sum_{i=1}^{3} |A|_i |C| \).

\( T \) is the transformation matrix which considers the equal displacements at the stiffener-plate junction has been presented in [15].

Finally, the element stiffness matrix of the stiffened plate is determined by:

\[
K_{ip}^{S_{1}} = K_{ip}^{S_{0}} + \sum_{i=1}^{n} K_{it}^{st}
\]

where \( K_{ip}^{S_{0}} \) is element stiffness matrix of plate and \( K_{it}^{st} \) are the opened stiffness matrix of the ith stiffener:

\[
K_{it}^{st} = \frac{V_{(15 \times 81)} T_{(15 \times 81)} K_{sp}^{st} T_{(15 \times 15)} V_{(15 \times 81)} - 1}{C_{2}}
\]

2.3. The mass matrix

The mass matrix of a stiffened plate element:

\[
M_{ip}^{S_{1}} = M_{ip}^{S_{0}} + \sum_{i=1}^{n} M_{it}^{st}
\]

where \( n_s \) is number of stiffener elements in the plate element; \( M_{ip}^{S_{0}} \) is the mass matrix of the plate; and \( M_{it}^{st} \) is the opened mass matrix of the ith stiffener.

\[
M_{it}^{st} = \frac{V_{(15 \times 81)} T_{(15 \times 81)} V_{(15 \times 15)} M_{st}^{st} T_{(15 \times 15)} V_{(15 \times 81)} - 1}{C_{2}}
\]

2.4. Governing equation

The equilibrium equation for an undamped stiffened structural system can be expressed as

\[
[M]\{Q\} + [K]\{Q\} = \{F\}
\]

where \([M],[K]\) are the assembled consistent mass and stiffness matrices, \([F]\) is the assembled nodal load vector, \([Q]\) and \([\bar{Q}]\) are the nodal displacement and acceleration vectors.

From the equilibrium Eq. (27), let \( \{F\} = 0 \) we obtain the equation of the free vibration problem as

\[
[M]\{Q\} + [K]\{Q\} = 0
\]

And the equation for static problem is obtained as follows:

\[
[K]\{Q\} = \{F\}
\]

3. Numerical results

3.1. Validation of the model

In order to check the reliability and accuracy of the present element, we consider the free vibration of clamped stiffened plates (Fig. 4) made of graphite/epoxy (ASI/3501-6) and studied by Dong-Min Lee. The geometry of the stiffened laminated plate are \( a \times b \times h_{p} = 500 \times 250 \times 1.04 \) (mm³), the lamination of plate is \((0/\pm 45/90\))°. The ply properties are \( E_{1} = 128 \) GPa; \( E_{2} = 11 \) GPa; \( G_{12} = G_{13} = 1.48 \) GPa; \( G_{23} = 1.53 \) GPa; \( v_{12} = 0.25 \); \( \rho = 1500 \) kg/m³ for both the plate and the stiffener. The stiffeners are the cross-ply laminated beam which has the same lamination ratio of \( 90/90\)° ply and perpendicular to the lamination of the plate. The thickness of the stiffener is \( t_{st} \), the height of stiffener is \( h_{tp} \). The five-first frequencies of the plates with different size of composite stiffeners are compared with those in [4] and shown in Table 1.

From Table 1, we can see that for the studied plate the natural frequencies of a plate with one Ox-stiffener are in good agreement with those of [4], in which the author used finite element method based on the first shear deformation theory with shear correction factor \( k = 5/6 \).

Five first natural frequencies calculated by our program are almost smaller than that in [4] because this based on third-order shear deformation theory. This theory no needs using the shear correction factor.

3.2. Free vibration of laminated stiffened glass fiber/polyester composite plates

In this study, composite materials in ship building are used: E-glass fibers and polyester resin. The plates with lamination \((0/90/0/90\)) reinforced by one Ox-stiffener of U, T or rectangular sections of equal cross-sectional areas are made for our free vibration tests like in Fig. 5. The flat plates are made out of four unidirectional plies and polyester; the stiffeners are made of glass fiber in mat and polyester.

Material properties of the plates and stiffeners are determined by experiment [16]: \( E_{1p} = 10.580 \) MPa; \( E_{2p} = 2640 \) MPa; \( G_{12p} = 1020 \) MPa; \( G_{23p} = 528 \) MPa; \( \rho_{p} = 0.19 \); \( \rho_{p} = 1600 \) kg/m³; \( X_{p} = 381.0 \) MPa; \( Y_{p} = 271.6 \) MPa; \( Y_{p} = 40.0 \) MPa; \( T_{p} = 20.0 \) MPa; \( S_{p} = 20.0 \) MPa; \( R_{p} = 145.2 \) MPa; \( E_{s} = 4807 \) MPa; \( E_{2s} = 4807 \) MPa; \( G_{12s} = 2054 \) MPa; \( G_{23s} = 961 \) MPa; \( Y_{s} = 0.17 \); \( \rho_{s} = 1400 \) kg/m³; \( X_{s} = 78.8 \) MPa; \( X_{s} = 142.9 \) MPa; \( Y_{s} = 78.8 \) MPa; \( Y_{s} = 84.7 \) MPa; \( T_{s} = 25.0 \) MPa; \( S_{s} = 25.0 \) MPa; \( R_{s} = 69.4 \) MPa. The plates and stiffeners are under different boundary conditions.

The plates and stiffeners are under different boundary conditions.

The results on three first natural frequencies for \( 6 \times 10 \) nine-node elements are compared with experimental ones in Tables 3 and 4. Free vibration modes of clamped composite plates with different shape of stiffeners are shown in Fig. 6.

| Table 1 Effect of size of stiffeners on free vibration frequencies of laminated stiffened composite plates. |
|---|---|---|---|---|---|
| Model no. | Size of stiffener \( t_{st} \times h_{st} \) | \( 1 \) | \( 2 \) | \( 3 \) | \( 4 \) |
| 1 | \( 0 \times 0 \) [4] | 85.1 | 134.0 | 207.4 | 216.1 | 252.5 |
| 2 | \( 1.56 \times 4.5 \) [4] | 108.3 | 207.3 | 214.9 | 252.3 | 329.2 |
| 3 | \( 2.06 \times 7.5 \) [4] | 170.6 | 209.2 | 257.7 | 292.9 | 338.4 |
| 4 | \( 3.64 \times 10.5 \) [4] | 168.4 | 207.2 | 253.4 | 287.9 | 335.5 |
| 5 | \( 5.20 \times 15.0 \) [4] | 227.8 | 270.2 | 294.5 | 321.8 | 373.7 |

Present 228.4 263.4 293.7 316.1 354.0

Present 212.2 225.3 268.8 308.6 352.0
From above mode shapes of stiffened plate with open and closed sections of stiffeners, we can see that three mode shapes of the plate reinforced by U-stiffener are different from those of the plate reinforced by T and rectangular stiffeners.

3.3. Experimental study on free vibration of laminated stiffened glass fiber/polyester composite plates

In order to validate above calculated results by our model, some experimental tests (Fig. 7) on free vibration were performed for those specimens which were shown in Section 3.2.

Three first natural frequencies of stiffened plates clamped at four edges and clamped at 2 \(O_y\) edges were measured by using a Multi-vibration measuring machine (DEWE BOOK-DASYLab 5.61.10) and are given in Tables 2 and 3.

Remark: \((\ldots\%)\) denotes the error percent between FE results and experimental ones.

From Tables 2 and 3, it is seen that natural frequencies obtained from numerical calculations are in good agreements with those of the experimental investigation, the difference ranges from 7.8\% to 15.6\% in case of the plates clamped at all four edges and from 5.9\% to 11\% in case of the plates clamped at 2 \(O_y\) edges and 2 other edges.

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Table 2
Natural frequencies (Hz) of stiffened glass fiber/polyester composite plates clamped at four edges.

<table>
<thead>
<tr>
<th>Mode no.</th>
<th>Rec stiffener</th>
<th>T-shaped stiffener</th>
<th>U-shaped stiffener</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Fem</td>
<td>Exp (%)</td>
<td>Fem</td>
</tr>
<tr>
<td>1</td>
<td>185.6</td>
<td>172.1</td>
<td>7.8</td>
</tr>
<tr>
<td>2</td>
<td>244.53</td>
<td>223.3</td>
<td>9.5</td>
</tr>
<tr>
<td>3</td>
<td>249.03</td>
<td>225.4</td>
<td>10.5</td>
</tr>
</tbody>
</table>

Table 3
Natural frequencies (Hz) of stiffened glass fiber/polyester composite plates clamped at 2 \(O_y\) edges and two other edges are free.

<table>
<thead>
<tr>
<th>Mode no.</th>
<th>Rec stiffener</th>
<th>T-shaped stiffener</th>
<th>U-shaped stiffener</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Fem</td>
<td>Exp (%)</td>
<td>Fem</td>
</tr>
<tr>
<td>1</td>
<td>41.4</td>
<td>39.1</td>
<td>5.9</td>
</tr>
<tr>
<td>2</td>
<td>52.2</td>
<td>48.0</td>
<td>8.8</td>
</tr>
<tr>
<td>3</td>
<td>87.1</td>
<td>79.9</td>
<td>9.0</td>
</tr>
</tbody>
</table>

Table 4
First ply failure loads and ultimate failure loads (MPa) of a stiffened plate with U, T and rectangular sections of stiffeners under uniformly distributed load.

<table>
<thead>
<tr>
<th>Boundary conditions</th>
<th>Simply support</th>
<th>Clamped on four edges</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stiffener profile</td>
<td>Rec.</td>
<td>T-shaped</td>
</tr>
<tr>
<td>FFL M. stress</td>
<td>0.4058</td>
<td>0.4126</td>
</tr>
<tr>
<td>Tsai-Wu stress</td>
<td>0.4070</td>
<td>0.4139</td>
</tr>
<tr>
<td>UFL M. stress</td>
<td>0.5635</td>
<td>0.7675</td>
</tr>
<tr>
<td>Tsai-Wu stress</td>
<td>0.5641</td>
<td>0.9200</td>
</tr>
<tr>
<td>Failed location</td>
<td>Top of layer 4; bottom of layer 1; mid of stiffener</td>
<td>Top of layer 4; bottom of layer 1; mid of stiffener</td>
</tr>
</tbody>
</table>
3.4. Progressive failure of stiffened laminated composite plates

Under a given bending load, the stresses at each integration point in the laminate are calculated in the material coordinate system. Then, the stresses are substituted into the failure criterion. If any failure occurs, the material properties of the laminate at that point are degraded according to the mode of failure as follows:

- Matrix failure: \( E_{22}, \nu_{12} \) are degraded.
- Shear failure: \( G_{12}, G_{23} \) and \( \nu_{12} \) are degraded.
- Fiber failure: \( E_{11}, E_{22}, G_{12}, G_{23} \) are degraded.

where \( E_{11} \) and \( E_{22} \) are the elastic moduli; \( G_{12}, G_{23} \) are the shear moduli; \( \nu_{12} \) is Poisson’s ratio.

The load is increased step-by-step, and the above analysis is repeated, until no additional lamina failure is detected. Finally after a ply-by-ply analysis, the ultimate failure load of a laminate is achieved.

Two failure criteria usually used for analysis of failure of composite structure are Maximum stress criterion and Tsai–Wu criterion.

3.4.1. Maximum stress criterion

In the maximum stress criterion, failure of any composite layer is assumed to occur if any one of the following conditions is satisfied:

\[
\sigma_1 > X_T \quad \sigma_2 > Y_T \quad \sigma_4 > R \quad \sigma_5 > S \quad \sigma_6 > T
\]

(30)

where \( \sigma_1, \sigma_2 \) are the normal stress components, \( \sigma_4, \sigma_5 \) and \( \sigma_6 \) are shear stress components, \( X_T, Y_T \) are the lamina tensile strengths in the 1, 2 directions and \( R, S, T \) are the shear strengths in the 23, 13 and 12 planes, respectively. When \( \sigma_1, \sigma_2 \) are of a compressive nature they should be compared with \( X_C, Y_C \), which are normal strengths in compression along the 1, 2 directions, respectively.

3.4.2. The Tsai–Wu criterion

According to this theory, failure of a composite layer is assumed to occur if the following condition is satisfied:

\[
\left( \frac{1}{\alpha_1 - \beta_1} \right) \sigma_1 + \left( \frac{1}{\alpha_2 - \beta_2} \right) \sigma_2 + \frac{1}{\alpha_1 \alpha_2} \sigma_1^2 + \frac{1}{\alpha_2 \alpha_3} \sigma_2^2 - \frac{1}{\sqrt{\alpha_1 \alpha_2 \alpha_3 \alpha_4 \alpha_5 \alpha_6}} \sigma_1 \sigma_2 \left( \frac{\nu}{\alpha_1} \right)^2 \geq 1
\]

(31)

In this example, we use two above criteria to calculate first-ply failure loads (FFL) and ultimate failure loads (UFL) of the stiffened plates which have considered in the example 2.

The plates and stiffeners are under different boundary conditions and subject to bending uniform load. The results on first-ply failure loads and ultimate loads are shown in Table 4.

It can be seen from Table 4 that after the initial failure, the loads can be increased by about from 38% to 86% in case of simply supported boundary condition and the load can be increased by about 110% in case of clamped at four edges.

The first-ply failure loads of the plates with U-stiffener are biggest and much more bigger than that of the plates with T or rectangular stiffener. The failure loads of the stiffened plates under clamped on four edges boundary condition are much bigger than ones under simply supported on four edges.

Failed position occurs at different locations depending on different types of sections of stiffeners.

4. Conclusion

Finite element model for analysis of laminated stiffened composite plates based on higher-order deformation theory have established. Experimental studies on vibration of laminated stiffened composite plates with different types of section of stiffeners are also performed. Based on the numerical and experimental results presented in this paper the following conclusions can be drawn:

- Experimental natural frequencies are in good agreement with those calculated by finite element model.
- For three studied glass fiber/polyester stiffened plates which have an equal volume of materials, the laminated stiffened plate with one U-stiffener shows bigger frequency than the plate with T and rectangular sections of stiffeners. Mode shapes of stiffened plate with U-stiffener are different with those of stiffened plate with T or rectangular stiffener.
- After the first-ply failure, laminated stiffened composite plates can carry out more and more additional bending load. Failed positions depend on the types of sections of stiffeners so it is noted when designing laminated stiffened composite plates.

Acknowledgement

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References